CASE STUDIES OF THE "MORE LIKELY" APPROACH TO PROBABILITY ASSESSMENT

Warren R Hughes of *hugheseconomics.com* – December 2024

INTRODUCTION

In this note, five case studies on the accuracy of the "more likely" approach to probability assessment are examined. For Case 1 the "more likely" methodology is reproduced in Table 1 with the final result taken to be the "correct" probabilities. Case 2 takes a slightly different approach. Case 3 fine tunes the Case 2 pairwise judgments to achieve greater accuracy. Case 4 considers another "correct" distribution examining the differences for basic and refined judgments. Case 5 includes an "equally likely" assessment.

CASE 1

Table 1 below shows the derivation of a "correct" distribution which the decision-maker (DM) uses judgment in an attempt to replicate. The better his/her judgments are, the closer the estimated distribution will be to the "correct" one. The pairwise ranges in this case are quite precise.

TABLE 1: CALCULATION OF PROBABILITIES FOR CASE 1

Scenarios		Pairwise Range		Probabilities				More Likely Values	
Events	**Ratios**	**Low**	**High**	**Low End**	**High End**	**Average**	**%**	**Average**	**%**
A	Base = 1	1.00	1.00	0.097	0.048	0.072	7	Base	Base
B	B/A	1.00	1.25	0.097	0.060	0.078	8	1.08	1.14
C	C/B	1.25	1.75	0.121	0.105	0.113	11	1.45	1.38
D	D/C	1.75	2.00	0.211	0.210	0.211	21	1.87	1.91
E	E/D	2.25	2.75	0.475	0.577	0.526	53	2.49	2.52
				1.001	**1.000**	**1.000**	**100**		

In estimating the distribution, the DM must assess first the order of event likelihood and then the "more likely" pairwise judgments as she/he moves through the pairwise assessments as in B over A etc. (ratios in Table 1). It is assumed that the DM has accurate judgment on the likelihood order from least to most likely. The real question then becomes how accurate are the pairwise judgments of the "more likely" character.

Table 2 below assumes that the judgments for the B, C and D events are of the 1 – 2 times "more likely" order and that for E/D is 2 – 3 times "more likely". The assumption that the DM can make judgments consistent with, but not as precise as, the "correct" judgments in Table 1 is not unreasonable. Resulting percentage differences from estimated to correct probabilities are of the order of 2% to 6% as outlined in Table 2.

TABLE 2: CORRECT AND ESTIMATED PROBABILITIES FOR CASE 1

	Correct Probabilities		Estimated Probabilities			Percent
Events	**P(·)**	**%**	**Pairwise**	**P(·)**	**%**	**Error**
A	0.072	7	Base 1 – 1	0.096	10	+3
B	0.078	8	1 - 2	0.109	11	+3
C	0.113	11	1 - 2	0.135	13	+2
D	0.211	21	1 - 2	0.186	19	-2
E	0.526	53	2 - 3	0.474	47	-6
	1.000	**100**		**1.000**	**100**	**0**

If the DM were able to fine-tune the three 1 to 2 pairwise judgments in Table 2 into low, medium and high values in the 1 to 2 range and also that for E/D in the 2 to 3 range (as in Table 1), then of course the estimated and correct distributions would converge. Although both distributions currently show a common pattern, this simulation indicates an overestimation in probabilities for the low-chance events, although not material. From this simulation, it might be concluded that the "more likely" procedure delivers probabilities within ±6% of the correct values.

CASE 2

In this simulation, the "correct" probabilities are taken as shown in Table 3. The "more likely" methodology uses the indicated pairwise ranges to derive the probabilities from these ranges. Again the DM is assumed to get integer ranges correct encompassing the correct "more likely" value with the results summarized in Table 3.

TABLE 3: CASE 2 CALCULATIONS

Events	Correct Percent Probability	More Likely Value	Pairwise Range	"More Likely" Probabilities	Percent Error
A	5	Base = 1	1 - 1	7	+2
B	7	1.40	1 - 2	9	+2
C	20	2.86	2 - 3	19	-1
D	30	1.50	1 - 2	26	-4
E	38	1.27	1 - 2	39	+1
	100			**100**	**0**

As for Case 1, the low chance event probabilities are overestimated but again not significantly. For this case, 4% is the maximum percentage error comparable with 6% for Case 1.

CASE 3

Case 2 is adjusted slightly by re-defining the C/B range to be 2.5 to 3.5. The idea here is that the DM uses a range of magnitude 1 encompassing the "more likely" value, which in this case is 2.86 or say 3. That is, the DM is capable of estimating correctly the approximate interval of magnitude 1 about the "correct" likelihood gain for this pairwise comparison, and it is only approximate. Here, a 2.5 – 3.5 range is better than the Case 2 range of 2 – 3 above. Results are summarized below in Table 4.

TABLE 4 : CASE 3 CALCULATIONS

Events	Correct Percent Probability	More Likely Value	Pairwise Range	"More Likely" Probabilities	Percent Error
A	5	Base = 1	1 - 1	6	+1
B	7	1.40	1 - 2	7	0
C	20	2.86	2.5 - 3.5	20	0
D	30	1.50	1 - 2	27	-3
E	38	1.27	1 - 2	40	+2
	100			**100**	**0**

The better judgment on the C/B range reduces the maximum error to -3% from -4%. Of course, better judgment makes for more accurate estimates. The assumption that the DM has this capability does not seem unreasonable. If the E/D pairwise judgment were further fine-tuned to 1 – 1.5, all estimated probabilities would be within ±1% of correct.

CASE 4

For Case 4, another distribution is examined for basic and more refined judgments as for Case 3 above. Results are documented in Table 5 below.

TABLE 5: CASE 4 CALCULATIONS

	Correct Percent	More Likely	Estimated Probabilities					
Events	Probability	Value	Pairwise 1	Percent	Error %	Pairwise 2	Percent	Error %
A	5	Base = 1	1 - 1	12	+7	1 - 1	5	0
B	10	2.00	1 - 2	13	+3	1.75 - 2.25	10	0
C	20	2.00	1 - 2	16	-4	1.75 - 2.25	20	0
D	25	1.25	1 - 2	23	-2	1 - 1.5	24	-1
E	40	1.60	1 - 2	36	-4	1.5 - 2	41	+1
	100			**100**	**0**		**100**	**0**

The Pairwise 1 values above are basic and are expected of the typical DM with a range of magnitude 1 about the "more likely" value. The pairwise 2 judgments are more refined with a narrower range of magnitude 0.5. As evidenced in Table 5, greater refinement of judgment, as for the Pairwise 2 values, makes for more accurate probability estimates within ±1% of correct.

CASE 5

Case 5 is similar to Case 4 but with an “equally likely” pairwise judgment. Results are documented in Table 6 below.

TABLE 6: CASE 5 CALCULATIONS

	Correct Percent	More Likely	Estimated Probabilities		
Events	**Probability**	**Value**	**Pairwise**	**Percent**	**Error %**
A	5	Base = 1	1 – 1	6	+1
B	10	2.00	1.5 – 2.5	10	0
C	20	2.00	1.5 – 2.5	19	-1
D	20	1.00	1 – 1	19	-1
E	45	2.25	2 – 3	46	+1
	100			**100**	**0**

Again we see a slight overestimation of the lowest probability but the estimations are very accurate to within ±1%.

RESULTS SUMMARY

In Table 7, the above results are summarized:

TABLE 7: SUMMARY OF ERROR PERCENTAGES

Case	Maximum Error %	I Average Error I %
Case 1	6	23.9
Case 2	4	17.9
Case 3	3	7.1
Case 4	7	41.6
Pairwise 2	1	7.3
Case 5	1	6.4

Table 7 shows that good judgment means individual percentage probabilities could be out by ±1% with an average percentage error for all estimates in a distribution of about 7%.

CONCLUSIONS

The case studies above indicate percentage errors of up to 7% magnitude from the correct percentage probabilities can be expected with the “more likely” methodology. But if judgment is more refined, then errors of only ±1% are possible. The above simulations suggest there could be a tendency for the “more likely” approach to probability assessment to overestimate the probabilities of the low chance events. More simulations are needed to confirm this aspect of the methodology. Good judgment and precision on the appropriate pairwise ranges will always improve the accuracy of the calculated probabilities for the “ballpark” distribution. And this can be further modified by the DM into a final distribution.