RANDOM THOUGHTS ON PROBABILITY ASSESSMENT

Dr Warren R Hughes
May 2024

The following ideas use the methodology outlined in the papers ***Assessing Probabilities of Unique Events: Calibrating Qualitative Likelihood Judgments into a Probability Distribution*** (2010), ***Assessing Probabilities for Events Pertaining to Buy/Sell and Similar Decisions*** (2017), ***Assessing Probabilities of Unique Events in Decision Making*** (2018), ***Assessing Probabilities of Unique Events: A New Approach*** (2019), ***Structuring Probability Assessments*** (2022) at doi.org/10.17265/1537-1506/2022.05.003, ***Thinking Probabilistically*** (2021) at doi.org/10.17265/1537-1506/2021.05.002 and ***Thinking Probabilistically Revisited*** (2022) at doi.org/10.17265/1537-1506/2022.01.001, ***A New Approach to Probability Assessment*** at doi.org/10.17265/1537-1506/2022.01.003, ***Probability Assessment*** at doi.org/10.17265/2328-2185/2022.05.006. All papers are available on the website at **hugheseconomics.com**.

Decision theorists claim that agent probabilities can be inferred from the choice of act from the alternatives available. It would be preferable to analyse the agent's or decision-maker's (DM's) intuitive probabilities initially and then determine the optimal act to implement based on maximising EV or EU.

The methodology outlined in all ***Assessing Probabilities*** papers (2010 – 2022) is one way of codifying judgment, intuition, hunches or "experience" using a systematic and exhaustive procedure over the entire sample space of all possible events. As with any creative venture that is assembled piece by piece (film, building etc.) the end result can only be fully appreciated on completion. This holds for the probability distribution resulting from a sequence of independent pairwise judgments. Only when a probability distribution is revealed in its entirety can the ultimate judgments on relative likelihoods be fully comprehended and, if necessary, amended.

Real-world problems involving prices of assets, commodities or goods can be assumed to generate at least 5 events or scenarios. First, price changes very little from the current price ($\pm \varepsilon$), price changes minimally ($\pm \alpha\varepsilon$) or price changes significantly ($\pm \beta\varepsilon$). Here α and β are chosen appropriately. If we knew for certain that the resulting price would be the current price plus $\beta\varepsilon$, then at the decision point, the correct decision would be "buy now" or "not sell yet." Of course, it cannot be known "for certain" what the resulting price will be although a decision must be made now. In this case, 4 pairwise assessments would be required to derive the actionable probability distribution.

In a recent book by Mohamed A. El-Erian, ***The Only Game in Town***, Random House 2016, he outlined three scenarios that PIMCO postulated related to the Lehman Brothers collapse in September 2008:

1. A Lehman takeover by a bank with a stronger balance sheet such as Barclays.
2. An orderly liquidation of Lehman as occurred with Long-Term Capital Management in 1998.
3. A disorderly liquidation.

The probabilities assigned by PIMCO to these scenarios were highest for a takeover and lowest for a disorderly liquidation, although the latter was in fact the scenario that eventuated.

In a recent paper, ***Assessing Probabilities of Unique Events in Decision Making*** in the Chinese Business Review, January 2018, Vol. 17 No. 1, 33 – 37, these probabilities are determined using the pairwise methodology outlined in previous papers.

In situations similar to the Lehman collapse it is possible to imagine a "good" scenario, a "bad" scenario and a "neutral" scenario that is neither "good" nor "bad." For three scenarios, the pairwise approach to probability determination is viable. Degrees of "good" and/or "bad" could be proposed to increase the number of possible scenarios beyond the three minimum.

Bayesian networks or decision trees can be used to spell out in detail the particular scenario under consideration. Then the pairwise relative likelihood procedure can be utilized to assign probabilities to uncertain outcomes.

There is nothing profound about the probability deriving process as outlined in the 2010 - 2022 papers. It is simply a methodical procedure to codify pairwise likelihood judgments into a probability distribution over a set of events pertaining to a unique situation. The qualitative judgments on pairwise comparisons are transformed into quantitative values enabling a probability distribution over all outcomes to be determined.

This initial probability distribution can then be "fine-tuned" to reflect the DM's definitive judgment based on a holistic view over all probabilities once the initial "ballpark" probability distribution is derived.

Wildly varying probabilities will lead to wildly varying values and possibly to market values in particular. This in turn leads to large windfall gains and losses generating calls for inefficient policies such as capital gains taxes. More effective probability assessment or generation would lead to "established", "acknowledged" or "recognised" probabilities resulting in less variation in expected values and/or "fair" prices and less opportunity for excessively profitable trading and/or windfall gains and losses.
.
Analysts state probabilities such as 30% chance of a double dip. But if we also knew their assessment of a major boom was 10%, we may not consider this to be "good judgment" and then discount their 30% call. We need to know their judgments over all possibilities. The holistic approach again!

A complete analysis requires the sample space over all events to include all possibilities no matter how unlikely currently. This allows Bayesian Revision to accommodate news that suddenly makes previously unlikely events more likely. Otherwise the analysis must be re-done to introduce previously excluded events.

After the initial probability distribution is determined, suppose betting odds are stated for one event or outcome at 3:1 on. This means that this event's probability becomes 0.75. Remaining event probabilities could be renormalized to a 0.25 total using the initial probabilities as weights. Of course, your judgment may be that the 0.75 call is not correct.

Bayesian Revision is most useful when new information arrives regularly. Situations such as mining with drilling tests and medical with patient tests would benefit from Bayesian Revision procedures.

The initial probability distribution derived by the DM would be "not invalidated" if another expert's odds for a "focal" or most likely event are "close to" the odds for the DM's distribution. For example, a 0.89 probability for an outcome is "not invalidated" by expert odds judgments from 7:1 on (0.875) out to 10:1 on (0.91). That is, a "buffer" of say ±3% (possibly as high as ± 5%) would allow reasonable variation in individual judgments yet still be within the same ballpark. See also remarks on convergence below.

Let CL(d) denote the compound likelihood of outcome d. Let a b c d e ... be a ranking of outcomes with outcome a the least likely up to ... Then if we have relative frequency information on b and d such that CL(d)/CL(b) is say 5.5 then the final probability distribution should reflect this fact. And pairwise likelihoods PL(c) and PL(d) will be constrained to reflect the 5.5 value also. Note that outcomes b and d may be regularly reported on whereas outcome c may be unique to the current decision situation. (See ***Incorporating Market Judgments into the Initial Probability Distribution*** below).

Once a prior distribution is derived (however rudimentary), successive new information would enable Bayesian Revision to converge on "correct" probabilities assuming successive "correct" judgments on consistencies with information for alternative SOWs (states of the world). See also remarks on convergence below.

Harold Jeffreys took a rather extreme position on this convergence of probabilities for different persons following accumulating evidence available to everybody. Let P(p|q) represent the probability of proposition p given data, evidence, knowledge or information q. Jeffreys writes:

> It is argued that because P(p|q) depends on both p and q it cannot be an objective statement, because different persons with different knowledge would assess different probabilities of p.

Jeffreys, however, goes on to write:

> ….two people both following the rules would arrive at the same value of P(p|q) …… They will arrive at consistent assessments if they tell each other their data and follow the rules.

Harold Jeffreys, ***Theory of Probability***, 3rd Edition 1961, corrected 1966, p. 406.

This suggests that different people having the same information through time and following agreed rules of inference or deduction (e.g., Bayes Rule) would converge on the same value for P(p|q). Above, it was suggested that probabilities within some range up to ±5% can still be considered "close enough" to be "agreed" probabilities or probabilities "not invalidated" by other evidence or judgment. Over time, we might expect the ±5% tolerance to shrink as conventional rules or revision procedures become more established.

In the information overload world of the 21st century, distinguishing critical new information from the repeated, recurrent, routine (or even mis-information) is a valued skill.

Suppose you are having difficulty calculating the “more likely” value between two events in the likelihood ranking. If possible, conceive an “imaginary” event or outcome having similar characteristics as the two real events with likelihood approximately halfway between those of the two real events. Proceed as normal but remove the “imaginary” event just prior to normalisation of the compound likelihoods retaining the pairwise likelihood of the “imaginary” event to calculate the compound likelihoods of the subsequent real events in the likelihood ranking.

From ***Fooled by Randomness*** by Nassim Nicholas Taleb:

> Probability in this book is tenaciously qualitative and literary as opposed to quantitative and “scientific.”
>
> Probability is not a mere computation of the odds on the dice or more complicated variants; it is acceptance of the lack of certainty in our knowledge and *the development of methods for dealing with our ignorance*, (original italics).
>
> ………….considering that alternative outcomes could have taken place, that the world could have been different, is the core of probabilistic thinking.

From ***The Black Swan*** by Nassim Nicholas Taleb

A Black Swan is an event with 3 attributes:

1. It is an outlier, as it lies outside the realm of regular expectations, because nothing in the past can convincingly point to its possibility.
2. It carries an extreme impact.
3. Human nature makes us concoct explanations for its occurrence *after* the fact.

De Finetti’s statement: Probability does not exist. According to Robert F Nau (***De Finetti Was Right: Probability Does Not Exist***, *Theory and Decision* 51: 89 – 124, 2001)…….. what de Finetti meant by this was that probability does not exist *objectively*, independently of the human mind. Rather, in de Finetti’s words:

> In the conception we follow and sustain here, only *subjective* probabilities exist – i.e., the *degree of belief* in the occurrence of an event attributed by a given person at a given instant and with a given set of information.

As interpreted by Itzhak Gilboa (***Theory of Decision Under Uncertainty***, CUP 2009, p 138)………de Finetti said that “Probability does not exist” meaning that it is not an objective feature of the world around us, but only a concept that is in our minds, something that we impose on reality around us, trying to make sense of our observations.

Over 75 years ago, Keynes also noted regarding probabilities for some events, “we simply do not know”. The following comments and quotes come from an article in the QJE in February 1937 reprinted in ***The Collected Writings of John Maynard Keynes, Vol. XIV, The General Theory and After: Part II, Defence and Development***, CUP 2013, p. 113. Keynes writes:

> The sense in which I am using the term [uncertain] is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence ………… About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know. Nevertheless, the necessity for action and for decision compels us as practical men to do the best to overlook this awkward fact and to behave exactly as we should if we had behind us a good Benthamite calculation of a series of prospective advantages and disadvantages, each multiplied by its appropriate probability, waiting to be summed.

Keynes goes on to offer techniques for dealing with probability estimation in these situations which today we would recognise as anticipating *efficient markets*, *rational expectations* as well as *the wisdom of crowds* as opposed to the judgments of a sole decision-maker.

Another interpretation of the Keynesian appraisal of the “don’t know” situation is the “gut feeling” decision such as between buy or wait, sell or hold etc. Based on the “gut feeling” a decision is made. The Keynesian assessment suggests a step-back procedure where all possible scenarios, outcomes or events relevant to the decision situation are enumerated. To each event, both a probability and a payoff can in theory be associated, even if they are only rudimentary. Does the resulting Benthamite calculation as in EV or EU endorse or reverse the "gut feeling" decision? Of course, most people do not do this as Keynes suggests, and go with their "gut feeling" if they have to. Procrastination, or no decision yet, may be a third option. A procedure for calculating the EV or EU in such situations is outlined in the 2017 paper above ***Assessing Probabilities***.

As noted above, the methodology outlined in the ***Assessing Probabilities*** papers is one way of codifying judgment, intuition, hunches or "experience ….." Alan Turing of WW II codebreaking fame had this to say about intuition:

> Mathematical reasoning may be regarded rather schematically as the exercise of a combination of two faculties, which we may call intuition and ingenuity. Intuition consists in making spontaneous judgments which are not the result of conscious trains of reasoning. These judgments are often but by no means invariably correct (leaving aside the question what is meant by "correct").
>
> "In pre-Godel times it was thought by some that it would probably be possible to carry this programme to such a point that all the intuitive judgments of mathematics could be replaced by a finite number of these rules. The necessity for intuition would then be entirely eliminated."

Turing saw the role of ingenuity as "aiding the intuition," not replacing it. Ingenuity may be replaceable today by modern search engines over all relevant aspects of the knowledge to be obtained.

Turing's Cathedral: The Origins of the Digital Universe by George Dyson, Pantheon Books, 2012, p. 252.

Much earlier in the 20th century, Keynes had similar ideas about probability formation:

> That part of our knowledge which we obtain directly, supplies the premises of that part which we obtain by argument. From these premises we seek to justify some degree of rational belief about all sorts of conclusions. We do this perceiving certain logical relations between the premises and the conclusions. The kind of rational belief which we *infer* in this manner is termed *probable* (or in the limit *certain*), and the logical relations by the perception of which it is obtained, we term *relations of probability*. The probability of a conclusion *a* derived from premises *h* we write *a*/*h*; and this symbol is of fundamental importance. (original italics).

Keynes goes on:

> The object of the theory or logic of probability is to systematise such processes of inference. In particular it aims at elucidating rules by means of which the probabilities of different arguments can be compared. It is of great practical importance to determine which of two conclusions is on the evidence more probable.

The Collected Writings of John Maynard Keynes, Vol. VIII, A Treatise on Probability, CUP 2013, p 121.

Turing's "rules" pay a role similar to Keynes' "relations" in the formation of probability judgments. This makes Jeffries' idea of an (objective ?) P(p|q), after sufficient publicly available evidence is assimilated by all, a probability about which all would agree. Or perhaps up to a value with small plus and minus variations and so only "objective" within a permissible range.

Technical analysis as currently practised by some security analysts could be regarded as a first tentative step in rule or relation construction at least as pertaining to the probabilities of future stock prices, exchange rates, commodity prices etc.

The 2010 article ***Assessing Probabilities*** defined the "more likely" classifications in Table 3 of that paper. For example, "slightly more likely" implied a "more likely" value in the range $1.0^{+} – 1.2^{-}$, just greater than the "equally likely" value of 1.0. Note that odds judgments could be used to derive these values. For example, at odds of 16:1 against (outsider ?) the probability of the event or winner is 1/17 or 0.0588. Slightly longer odds for an alternative event or winner at 17:1 imply the 16:1 shot is 18/17 or 1.06 "more likely"; odds of 25:1 imply a 1.53 "more likely" value for the 16:1 shot (i.e., 26/17); and odds of 33:1 imply the 16:1 shot is 34/17 or twice as likely.

While the equivalences between assessment using odds or the "more likely" criterion can be explored, the "more likely" approach is preferred since it can be used more generally than the more explicit odds approach. As outlined above, odds require a numeric assessment such as a 5:1 event being only a 1 in 6 chance or requiring at least 6 replications of the same situation for an expected 1 occurrence of the event in question. The "more likely" approach is less precise but applicable in a situation where it is believed event B is “slightly more likely” than event A. This belief can be due to the current climate, sentiment, outlook or other prevailing mood justifying a “more likely” value in the range 1.0^{+} - 1.2^{-}. The reason for the “slightly more likely” belief cannot be articulated to the degree that A is at 7:1 whereas B is at 6:1 (a “more likely” value of 1.14) whereas a chosen value in the 1.0^{+} - 1.2^{-} range acceptably reflects current beliefs and, with other pairwise judgments, results in a verifiable probability distribution.

An historical use of classifying degrees of certainty or confidence is reported in the book ***Codebreakers: The Inside story of Bletchley Park*** edited by F.H. Hinsley and Alan Stripp, OUP 1993, pages 20 – 21. The reliability of a decode was indicated by a percentage qualifier with a defined meaning as follows:

B% some margin of doubt
C% caution if not confirmed
D% largely guesswork

A definition for A% was not provided but presumably this would be “certain” or at least “almost certain.”

In his recent book ***The End of Theory*** (Princeton University Press, 2017), Richard Bookstaber introduces four concepts having implications for economic modelling. These are emergent phenomena, non-ergodicity, radical uncertainty and computational irreducibility. He also reviews the George Soros concept of reflexivity. His overall conclusion is that the complexity resulting for economic modelling from all the above means “we don’t even know what the things are to which we should be assigning a probability” (page 65). This echoes the Keynes’ comment “we simply do not know.” Nevertheless, as Keynes concluded, decisions and actions must be undertaken. The “more likely” approach using numbers as outlined in the papers mentioned above is at least feasible (even if the numbers are tentative or exploratory) whereas trying to directly form explicit odds or probabilities for each event individually over all possible events may be impossible as suggested by Keynes and Bookstaber.

The methodology outlined in the 2010 – 2022 papers would be especially appropriate for the formation of a prior distribution over a set of events pertaining to a unique situation as in the Lehman failure. In such a case there is no “objective” information of past events to determine probabilities. The basis for forming probabilities is the experience or intuition of the DM plus any news, hearsay, rumours etc. on the situation under consideration. In this case, rudimentary primary judgments of a “more likely” nature can yield an actual prior distribution that can then be amended to reflect the DM’s ultimate judgments on the relative likelihoods.

In the following analysis, the relationship between odds formation and the “more likely” approach to probability determination is explored. Let us assume 3 events one of which must occur. The events are A, B and C where A is least likely and C is most likely. Let the odds against these events be respectively: 3:1, 2:1 and 3:2. The following table derives the probabilities based on these odds judgments.

Probability Derivation from Odds Judgments

Events	Likelihood	Odds Against	Odds Probabilities	Adjusted Probabilities
A	Least likely	3:1	0.25	0.25
B	Intermediate likelihood	2:1	0.33	0.34
C	Most likely	3:2	0.40	0.41
			0.98	**1.00**

Note that the probabilities of B and C have been increased slightly in accordance with the axioms of probability. Given the rudimentary nature of the “more likely” judgments to be examined below, it is thought appropriate to report all calculations to 2 decimals. This allows for the commonly accepted approach of expressing probabilities as percentages (with event C having a 41% chance as derived above) and eliminates any charge of spurious accuracy. The “more likely” equivalent to the above probability derivation is outlined in the following table.

Probability Derivation using the “More Likely” Approach

Events	Odds Against	# of Trials for 2 Occurrences	Pairwise More Likely Value	Compound Likelihood	Probabilities	Final More Likely Value
A	3:1	8	1.00	1.00 = 15/15	15/59 = 0.254 ~ 0.25	1.00
B	2:1	6	8/6 =1.33	20/15	20/59 = 0.339 ~ 0.34	1.36
C	3:2	5	6/5 = 1.20	24/15	24/59 = 0.407 ~ 0.41	1.21
				59/15	1.00	

Exactly the same probability distribution is derived using the Odds and the “More Likely” approaches. The main conclusion here is that whereas Odds judgments result from specialised knowledge, the approach using the “More Likely” or primary judgments can be utilised by anyone. If the primary judgments are misguided or bad, then the resulting probability distribution will reflect these bad judgments, but the calculations are not difficult for both good and bad judgments. The rudimentary nature of the pairwise “more likely” values could mean that such judgments increment above 1.00 using 0% (equally likely), 5%, 10%, 15% etc. rather than utilising values like 17% or 96%.Of course, specific known relative frequency data for two events could result in a 17% or 96% more likely judgment.

Since the primary judgments (PJs) may be “vague” or “rudimentary,” differences between the original PJs and the final “More Likely” values (as shown above) could show acceptable variation of say up to ± 5%. For example, 1.36/1.33 from the above table shows a 2.3% variance for event B. And of course, as noted above, when the final distribution is being adjusted, the DM may change his/her mind as to what the final probability values should be. These final judgments are definitive and may not necessarily agree exactly with the original PJs which are really only a means to the end of an initial or first pass probability distribution utilizing all information available at that time.

In the absence of a standard measuring device such as a tape or a weighing machine, the less likely event in the pairwise comparison serves as the “standard.” The more likely event then is assessed by a number greater or equal to one according to its assessed greater (or equal) likelihood.

If there are quoted odds for one or more events, the pairwise methodology can be used for remaining events suitably normalised, so all probabilities sum to one.

Using Saaty’s AHP approach, a perfectly consistent reciprocal matrix can be formed from the number of trials for 2 occurrences shown in the table above. The reciprocal matrix, resulting AHP priorities and probability distribution are detailed below.

A COMPLETE ANALYTIC HIERARCHY PROCESS (AHP) ANALYSIS

Reciprocal Matrix				**Event**	**Priorities**	**Probabilities**
Event	**A**	**B**	**C**			
A	1	6/8	5/8	**A**	0.2542	0.25
B	8/6	1	5/6	**B**	0.3390	0.34
C	8/5	6/5	1	**C**	0.4068	0.41
				Totals	**1.0000**	**1.00**

Under a complete AHP analysis, the probability distribution is the same as derived above. Note that the A, B, C values for the respective Priorities can be derived using the geometric mean for each row of the Reciprocal Matrix and normalizing the sum to unity. The resulting probabilities (or priorities) are identical to those derived using Saaty’s principal eigenvalue method. Modern calculators can directly estimate eigenvalues and eigenvectors for matrices of modest dimension.

Brunelli (***Introduction to the Analytic Hierarchy Process***, Springer 2015, p. 19) shows how the geometric mean priorities can be interpreted as the result of an optimization problem minimizing a squared error objective function.

A complete AHP analysis requires n(n-1)/2 pairwise values or 45 judgments for a 10-event situation. The adjacent-events only judgments total (n-1) or 9. The additional judgments if accurate, could improve the “correctness” of the initial probability distribution or could prove redundant if (almost) perfectly consistent with the initial (n-1) adjacent-event pairwise values. Accordingly, the smaller number of initial judgments may be adequate for deriving the initial “ballpark” probability distribution prior to determining the final probabilities.

ECONOMISING ON THE CALCULATIONS

For n events, outcomes or scenarios, (n-1) pairwise comparisons suffice to determine a "ballpark" distribution. A full AHP analysis (using the principal eigenvector or geometric means) requires n(n-1)/2 pairwise judgments. The alternative procedures can best be compared using a 3 x 3 matrix for events A, B and C with a B over A (B/A) more likely value of 2 and a C/B value of 3. Hence a perfectly consistent C/A value of 2 x 3 = 6.

ECONOMICAL COMPUTATION OF PROBABILITIES

Event	Pairwise Value	Compound Value	Probabilities
A	1.00	1.00	1/9 = 0.1111
B	2.00	1.00 x 2.00 = 2.00	2/9 = 0.2222
C	3.00	2.00 x 3.00 = 6.00	6/9 =0.6667
		9.00	**1.0000**

In matrix form, the calculations would be as follows:

COMPLETE AHP CALCULATIONS

Reciprocal Matrix				Probabilities		
Event	**A**	**B**	**C**	**C/A = 5**	**C/A = 6**	**C/A = 7**
A	1	1/2	A/C	0.1220	0.1111	0.1025
B	2	1	1/3	0.2297	0.2222	0.2158
C	C/A	3	1	0.6483	0.6667	0.6817
				1.0000	**1.0000**	**1.0000**

A perfectly consistent C/A value of 6 results in the same probabilities as above. A higher C/A value lowers (raises) the probability of A (C) slightly. A lower C/A value of 5 does the reverse. This illustrates the tension between the economy of calculation by the minimal (n-1) judgments versus the potentially more accurate distribution with a greater number of pairwise judgments (more information). More judgments, employing the practice makes perfect axiom, may improve the accuracy of the probability distribution, although we have no way of ascertaining what "accuracy" means here. Conversely, the above example shows a maximum difference in probabilities of ± 0.03 to 2 decimals, possibly not great enough to justify the extra effort on the part of the DM given the objective of a "ballpark" distribution. For a 5-event problem, the number of pairwise judgments would be 4 and 10, respectively.

INCORPORATING MARKET JUDGMENTS INTO THE INITIAL PROBABILITY DISTRIBUTION

The methodology in this case is best illustrated using a 4-event situation where the initial likelihood ordering is A, B, C and D (most likely). Suppose quoted market odds are 4:1 for B and 3:2 for D. Hence event D is twice as likely as B according to the market's judgment.

Let BA denote the DM's "more likely" value for B over A at 1.25. Similarly, let CB be the "more likely" value for C over B and DC the final "more likely" judgment of the DM for D over C.

Event	Compound Likelihood
A	1.0
B	1.25
C	CB*1.25
D	DC*CB*1.25

To satisfy the market's odds information we must have: (DC*CB*1.25)/1.25 = DC*CB = 2.0

Accordingly, the DM's final judgments must satisfy this constraint to conform with the market's odds. Hence if CB is assessed at 1.35 then DC necessarily equals 1.48 using 2 decimal values (or 1.50). With either of these values the methodology produces a probability distribution of 0.16, 0.19, 0.26 and 0.39 for A, B, C and D respectively. With adjustments, the resulting distribution could exactly reflect the market's odds judgments. Of course, the DM may not agree with the market's stated odds for B and D leading to a different initial probability distribution.

Similarly, if there were a single market odds quote for event C at 3:1 (probability 0.25), the constraint becomes for BA at 1.25 and given DC:

CB = 9/(15 – 5DC) or DC = 3 – 9/5CB

As a result of this constraint, some CB, DC combinations become inadmissible given the 1.25 judgment for BA. The following table shows that as the DC value drops from 1.50 down to 1.30 (with corresponding changes in the CB value), the probability of event D drops from 0.38 down to 0.32.

PROBABILITIES FOR VARYING DC & CB VALUES WITH P(C) FIXED AT 0.25

EVENT	**DC = 1.50 CB = 1.20**	**DC = 1.45 CB = 1.16**	**DC = 1.40 CB = 1.125**	**DC = 1.35 CB = 1.09**	**DC = 1.30 CB = 1.06**
A	0.166 667	0.172 340	0.177 778	0.183 423	0.188 768
B	0.208 333	0.215 424	0.222 222	0.229 279	0.235 961
C	0.25	0.249 892	0.25	0.249 914	0.250 118
D	0.375	0.362 344	0.35	0.337 384	0.325 153
TOTAL	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

Note that a 12% drop in the CB “more likely” value results in a 13% rise in P(B). Similarly, a 13% drop in the DC “more likely” value results in a 13% decline in P(D), using 6 decimal probabilities.

Of course, irrespective of market odds, the DM could impose a constraint such as P(C) = 0.25 on the procedure and then derive the remaining probabilities conditional on this judgment.

CONCLUSIONS

In assessing probabilities over several events, estimating each event’s probability in turn is flawed. Probabilities of events other than the one being assessed may be considered but only as a group. The final event assessed is in effect a “residual” probability as in one minus the sum of the probabilities of the previously assessed events.

The pairwise procedure outlined above more specifically compares respective likelihoods with at least two other events, the least and most likely events excepted. And in a complete AHP analysis, every event is compared with every other event. Any difficulties as with events with widely dispersed likelihoods can be circumvented with a reduced number of pairwise comparisons over all events as outlined above. In both cases, several pairwise comparisons determine the final probability distribution, and no probability is determined as a “residual” probability.

A group discussion on an appropriate pairwise value for 2 events should ensure that all relevant issues are raised leading to an appropriate value choice. And sensitivity analysis is now easily handled with spreadsheets to examine all possibilities. A group discussing a pairwise value might be compared to a group deciding on the value of an asset. The value will be discussed from all possible angles and therefore result in a more accurate value. This could be labelled the “wisdom of crowds aspect.”

Each event has certain attributes, properties, factors, causes etc. that interact and in totality determine its estimated likelihood in the eyes of the DM. The event it is compared to possesses these same attributes etc. perhaps to different degrees leading to a different estimated likelihood for the second event and a relative gain or loss in likelihood over the first event.

It may be envisaged theoretically that it is possible to delineate all such causes and analytically assess their individual importance for a likelihood estimate in a particular case. This could be a full-blown AHP analysis with a sophisticated hierarchy of cause and effect leading to resulting likelihoods for the respective events. The methodology outlined here assumes this to be beyond the capabilities of the DM but that he/she can intuitively synthesize this myriad of causations into an overall percentage gain in likelihood for one event over another in a holistic pairwise comparison. Hence there results a percentage judgment that one event is say 10% (slightly), or 50% (significantly) or 90%+ (substantially) etc. “more likely” than the other event.

Even if the DM uses an incomplete and/or incorrect probability assessment framework for the “more likely” judgments, he/she may still fortuitously approach the correct ***relative*** likelihoods in a series of pairwise comparisons resulting in a “correct” final distribution over all events. That is, faulty or incomplete reasoning may still be useful if it is applied consistently! Currently, with the same cognitive limitations, DMs must undertake the harder task of estimating probabilities directly. The methodology outlined here requires similar but less demanding judgments in the first instance.

As reported above, Nassim Taleb in ***Fooled by Randomness*** (2nd Edition 2005, Random House: p. x) writes "considering that alternative outcomes could have taken place, that the world could have been different, is the core of probabilistic reasoning". But in the real world, people do not think in probabilistic terms because it is too difficult to estimate probabilities of different outcomes or scenarios. It is more convenient to act as if one of the scenarios is *certain* to happen. If we could systematically formulate the judgments in our head as to what is "more likely" between two alternatives in an easy fashion, we would be motivated to do just that. If a procedure as illustrated above is followed, eventually a series of such pairwise judgments can be formulated into an initial probability distribution. This distribution is the "ballpark" within which final probability judgments can be arrived at. And this distribution can lead to reconsiderations of what is in fact "more likely" (and the extent of such) prompting a revised probability distribution defining the "ballpark." Eventually, any recursion of judgments-to-probabilities will lead to a FINAL probability distribution that the DM is comfortable with.

A probability distribution can only be "correct" if it accurately reflects the considered judgments of the DM. By definition, the final distribution of the DM must be correct since he/she must always agree with the final values determined or would change them otherwise. At the very least, the final values are "close enough" for current time-constrained decision making.

Enumerating ALL possibilities (current event descriptions must reflect the ALL) means the resulting outcome must be one of them. Therefore, good judgment comes down to "What was its probability?" Consistently high probabilities for resulting outcomes means the DM shows "good judgment" in chance situations. But even if a low probability event does occur, the DM has presumably thought ahead of time about his/her actions in that eventuality.

Only when a complete probability distribution is appraised do certain factors become apparent. For example, a distribution may be weighted too heavily in favour of good (or bad) outcomes in the DM's re-considered judgment. Readjustments can then be made in light of the complete picture with an axiomatically correct distribution.

The initial pairwise judgments define the "ballpark" distribution but are not sacrosanct. In deciding on a very low probability event, we know already that this is only say a 5% or less chance. The "more likely" value then is going to be 5 - 10 – 15 or more, and it does not really matter what the final value is as long as a low probability results for this event. When, however, we consider two events that are closer together in likelihood, we may be undecided as to use 1.15 or 1.35 to reflect "a little more likely" difference. Here, 1.25 gets us to the "ballpark." Then other information (as in a say no more than a 5% difference between these events) enables us to fine-tune the final distribution. The final "more likely" pairwise values may end up quite different from the initial "ballpark" values. Exact matching between the initial and final pairwise values is not to be expected. And recursion between "more likely" pairwise judgments and the "ballpark" distribution may enhance the derivation of a more "accurate" final distribution.

Revision of probabilities in the light of new information may be easier if there is a probability distribution, however crude or sketchy, to start with. Alternatively, revising with only unformalized likelihood notions on possible events may not be easy.

For the pairwise judgments it could be asserted that a set of common factors pertains to each pairwise judgment in turn. In reality, it may be that a certain pairwise judgment highlights factors that may be missing (or of lesser effect) from the other pairwise judgments. Should this be of concern? Possibly not with the averaging process (either by the eigenvector method or geometric mean) over all pairwise judgments sufficing to incorporate all relevant factors at some point in determining the final distribution. There is a trade-off between economy of method with the minimal number of judgments versus ensuring all factors are accounted for (if not to the same extent in every pairwise judgment) in determining the final distribution. Since each pairwise judgment may incorporate factors unique to that particular judgment, this could be an argument for requiring the full n(n-1)/2 judgments as opposed to the minimal (n-1) judgments.

For example, in the POTUS 2024 election, comparisons involving Kennedy need to take into account that no Independent has won the Presidency since 1856 when Republicans became the second major party. But this factor is not so relevant in the Biden versus Trump comparison.

Probability determination in the mind of the DM processes information in a way that may not be easy to replicate in an algorithm. In this new approach (outlined in the paper ***Probability Assessment***) the DM summarizes his/her processing with a range (low to high) of "more likely" values in comparing two events (pairwise values). This range or spread may be narrow or wide. Candidate distributions can then be calculated via a spreadsheet followed by routine methodology to axiomatically correct probability determination. The resulting distribution could be utilized immediately or further developed using other information triggered by the analysis carried out. For example, assessing probabilities on groups of events as in “good” or “bad” outcomes. This could guard against wishful thinking or undue pessimism. Apart from the initial range determination on “more likely” values, all calculations are routine with spreadsheet methodology. Wide spreads in “more likely” values initially can be narrowed with final calculations refined from preceding calculations. The simplicity of the technique (especially with the minimal calculations) invites exploration by anyone in using it. With this perspective, probability determination may be better thought of as an iterative process (oscillating between probabilities and “more likely” ranges) rather than a methodology which in one pass of calculation produces the ”correct” or final distribution.

The above is not a revolutionary algorithm to calculate exact or precise probabilities, but an aid to better decision-making that specifically incorporates uncertainty. Methodologies, as outlined here, may help as in producing interim probabilities that are further refined with further information not used in the lead-up calculations. Searching for a “correct” probability distribution could end up like discovering pornography – he or she recognizes it when it appears.

Two broad strategies for probability determination have been suggested. In the first, n(n-1)/2 pairwise judgments would ensure that all relevant aspects of the situation under analysis are covered by the DM. Averaging of any resulting inconsistencies in these judgments is then necessary. Alternatively, minimal calculation can be used with oscillation between proposed pairwise ranges and resulting probabilities with a discovery of the final “correct” distribution when it appears.

Word Count 6456, 23rd May 2024