

Variations in Probability Assessment With Atypical Scenarios

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Probability assessment in some scenarios may involve unusual aspects such as requiring certain values for some events and extremely high or low probabilities in other cases.

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Introduction

Probability assessment using the “more likely” methodology has been outlined in Hughes (2022a) and Hughes (2022b). The pairwise approach to measurement used in this methodology is well documented in Saaty (2008). This note explains some aspects of the procedures in more detail, in particular, accounting for almost certain events with large values for the pairwise range and requiring specific probabilities for certain events.

Basic “More Likely” Calculations

The table below illustrates the basic “more likely” range or pairwise judgments and resulting probabilities. Four events are ranked as follows:

- Base: Least likely event in the assessment.
- Sli: A slightly more likely event than Base.
- Mod: A moderately more likely event than Sli.
- Sig: A significantly more likely event than Mod.

Table 1

Pairwise Ranges With Probabilities Illustrating the “More Likely” Approach

Event	Pairwise ranges		Probabilities				More likely values	
	Low	High	Low	High	Average	Percent	Average	Percent
Base	1.00	1.00	0.184	0.113	0.149	15	Base	Base
Sli	1.00	1.25	0.184	0.142	0.163	16	1.09	1.07
Mod	1.25	1.75	0.230	0.248	0.239	24	1.47	1.50
Sig	1.75	2.00	0.402	0.496	0.449	45	1.88	1.88
			1.000	0.999	1.000	100		

Note that the low distribution increases the probabilities of the low chance events at the expense of the high chance events. The high distribution does the opposite. Using the calculations offered by the three distributions, the decision-maker (DM) has plenty of options to consider in revising the judgments and deciding on the final distribution. Revising a complete distribution (however rudimentary) may be easier than event-by-event probability revisions.

Extremely “More Likely” Events

To illustrate an aspect of the methodology, we now introduce a fifth event (Ext) which is extremely more likely than event Sig at say 4-5 times “more likely”. When events in the likelihood ranking are judged, this much “more likely” the DM possibly has only imprecise reasoning to justify these numbers which may be limited to the fact that this more likely event is substantially greater in likelihood than its alternative. Integer ranges are then preferred at a level the DM judges appropriate. For a 4-5 times “more likely” judgement, calculations are shown below in Table 2.

Table 2

Probabilities for Five Events Including a New Extremely “More Likely” Event (Ext)

Event	Pairwise ranges		Probabilities				More likely values	
	Low	High	Low	High	Average	Percent	Average	Percent
Base	1.00	1.00	0.070	0.032	0.051	5	Base	Base
Sli	1.00	1.25	0.070	0.041	0.056	6	1.10	1.20
Mod	1.25	1.75	0.088	0.071	0.080	8	1.43	1.33
Sig	1.75	2.00	0.154	0.142	0.148	15	1.85	1.88
Ext	4.00	5.00	0.617	0.713	0.665	66	4.49	4.40
			0.999	0.999	1.000	100		

The new event Ext captures two thirds of the total likelihood with the initial four lower chance events reflecting the same pattern as previously with probabilities approximately one third of their Table 1 values. Looking at the results, the DM may consider the 66% probability for P(Ext) too high, resulting from the (possibly arbitrary) range of the 4-5 times “more likely” judgment. Halving the range to say 2-3 results in a 7%, 8%, 12%, 21%, 52% distribution with a significantly lower probability for event Ext. Spreadsheets make for easily revised calculations.

Incorporating Conditions Into the Probability Assessment

When the DM has other pertinent information on the probabilities, this can be incorporated into the methodology as demonstrated below. The example uses a three-event problem but is easily extended to more events, as necessary.

Table 3

Three-Event Problem Approach

Events	Pairwise ratio	Pairwise values	
		Low	High
A	Base = 1	1	1
B	B/A	x	2x
C	C/B	2	3

Table 3 shows the DM ranking the events from least to most likely as in A, B, and C. It is assumed here that he/she has good ideas about the C/B pairwise value but less idea about the B/A value, but it could be the other way around (see below). The x-2x values allow for one unknown to be determined by one condition using other DM information. Here this condition is that the most likely outcome C has probability of 60% or more. Accordingly, the combined probabilities for events A and B must total 40% or less with x to be determined by this condition. Specified pairwise values as above must, of course, be consistent with the relatively high probability required for event C in this case. In principle, each unknown in the analysis will require a condition

on the probabilities in addition to the original pairwise values. Using a range as in x to $2x$ means there is just one unknown and simplifies the algebra. In practice, the speed and ease of spreadsheet re-calculation substitutes for precise, algebraic analysis, which is not reproduced here, but would use the information in Table 3.

Basic algebra determines an x value of 1.2 in this case. Percentage probability results for various values are shown in Table 4 and these should suffice for routine decision-making as in buy, sell, or hold.

Table 4

Sensitivity of Probabilities to the $P(C) \geq 60\%$ Condition

Events	Pairwise ranges	Percentage probabilities			More likely values		
		$x = 1.2$	$x = 1.5$	$x = 2$	$x = 1.2$	$x = 1.5$	$x = 2$
A	Base: 1-1	16	13	10	Base	Base	Base
B	$x-2x$	24	25	26	1.50	1.92	2.60
C	2-3	60	62	64	2.50	2.48	2.46
		100	100	100			

Once the x value is determined as in 1.2 above, the DM can decide on an appropriate pairwise range incorporating this value. If the unknown range in Table 3 was expressed as x to $(x + 1)$, the x value in this case becomes 1.24 with the same distribution as for $x = 1.2$ above. Note too that if the B/A range was 2-3 and the conditional C/B range was x to $(x + 1)$, the required x value becomes 1.7 with a resulting distribution of 12%, 28%, and 60%.

Almost Certain Events

As the recent UK election in July 2024 demonstrated, “almost certain” events may manifest themselves. In this case it was a clear Labour victory. A 90%+ probability may be justified for the “almost certain” event. This can be approximated using the formula for a 0.95 probability as follows:

$$x / (x + n - 1) = 0.95 \text{ which for } n = 5 \text{ events becomes } x = 76.$$

This formula assumes events other than the “almost certain” event are equally likely, which may not be the case but suffices to determine the high value ranges necessary in situations like this. Here the range required could be say 70-80. Variations around this range may be needed before the DM settles on a final distribution. With an “almost certain” event at say 95%, the final distribution could, of course, be assessed directly with 5% allocated over the remaining events with no range analysis needed. A very low probability for an event may also require appropriate high value ranges.

Conclusions

Using mid-points of ranges reduces calculations and produces distributions almost identical to the average in the above tables—maybe 1% to 2% changes if any. The low and high distributions, however, may be useful in deciding on a final distribution. Considerations as above may be pertinent for the “more likely” methodology of probability assessment. Once a “ballpark” distribution is determined, the ease, speed, and accuracy of spreadsheet calculation makes iterative trial-and-error a viable approach to ultimate probability assessment.

References

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